Generalised and fractional Langevin equations-implications for energy balance models

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Energy Balance Models (EBMs) have a long heritage in climate science for modelling global mean temperature anomalies. Many types of EBM have now been studied, including examination of spatiotemporal, and particularly latitudinal dependence, as well as possible low dimensional effects. Of particular interest to the statistical mechanics community is the stochastic EBM, which allows direct treatment of climate fluctuations and noise. Some recent stochastic EBMs (e.g. [1]) map on to the Langevin equation, with temperature anomaly replacing velocity, and other corresponding replacements being made. This raises the question as to how far the full technology of modern Langevin modelling could be applied to this aspect of the climate problem.

This talk will discuss this question, and give some preliminary answers. We first note that the most familiar Langevin equation is a limiting case of the generalised Langevin equation (GLE) which so far has seen little or no application in climate science, raising the question of whether a GLE-type EBM can be derived or motivated. This is particularly important because, as noted in [2], the EBM studied by Padilla et al [1] used a correlated red noise term but has a constant dissipation term. A fluctuation-dissipation theorem is thus precluded by its construction.

We then note that long memory simplifies the GLE to a fractional Langevin equation (FLE). The still controversial experimental evidence for long range memory in global temperature has already motivated investigation of a power law response model [3,4]. We go beyond this to ask whether an FLE-type EBM exists, and what its solutions would be.

See also these talks: Klages, Anomalous Fluctuation Relations, Newton Institute, October, 2013; Watkins, Generalised and Fractional Langevin Equations-Implications for Energy Balance Models, Norklima Meeting, Tromso, September, 2014 ; Klages, Anomalous Langevin Dynamics, Fluctuation Dissipation Relations and Fluctuation Relations, Climathnet meeting, Dartington Hall, January, 2015.

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